NTIA REPORT 01-383

The Temporal and Spectral Characteristics of Ultrawideband Signals

William A. Kissick, Editor

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EXECUTIVE SUMMARY

Objectives of this Work

A preliminary objective of this study was to develop a description and gain an understanding of the ultrawideband (UWB) signal structure based on current, and hopefully typical, UWB system capabilities and applications. This began with a determination, from specifications and/or direct measurement, of the salient temporal characteristics of UWB signals that included minimal descriptions of their modulation schemes for data and/or voice and detailed descriptions of their pulse shape, width, repetition rate, dithering, and gating characteristics. Then, key fundamental aspects of UWB signal behavior were derived from first principles. This provided a basis for identifying what to measure and the effects certain temporal characteristics have on the spectral characteristics.

The primary objective was to observe and record the temporal and spectral characteristics of various UWB signals using both highly accurate measurement methods and practical approaches with commercial off-the-shelf (COTS) test equipment. The measurements are supported by the theoretical work noted above and confirmed through simulation. Meeting the primary objective has provided the technical information needed by NTIA to develop policies for use of UWB by the Federal government and to work with the Federal Communications Commission (FCC) to develop rules and regulations for UWB emissions. Secondary objectives included the development and description of reliable and repeatable measurement methods using COTS test equipment, and measurement of the effects UWB signals have on several, selected Federal Aviation Administration (FAA) radar systems.

There are unanswered questions and claims regarding UWB. Some say that the 2-GHz bandwidth (nominal, based on a pulse width of 1 ns) is ideal in many applications because the already low total power of a UWB signal is spread so "thinly" that the spectral power density in any conventional (bandwidth limited) channel is inconsequential. The claim goes further to say that the signal is similar to Gaussian, or white, noise and therefore it is like the background noise any communications or radar receiver experiences.

UWB Technology and the Radio Spectrum

UWB technology may offer very effective solutions for various communications and sensing applications; but its uncommon approach of using narrow pulses, or impulses, as a basic signal structure rather than generating and modulating a sinusoidal carrier results in an unusually wide emission bandwidth. Since such a wide signal covers many radio bands and services, the conditions under which it can operate without causing undue interference must be determined before UWB systems are allowed to proliferate.

The use of a carrier signal by nearly all existing services that share the radio spectrum helps ensure that the bandwidth of the emissions of those signals can be kept as narrow as possible for any given application, i.e., the bandwidth required to transmit the information of interest or perform the necessary sensing functions. This approach allows for effective and efficient spectrum management and frequency assignment procedures for sharing of the radio spectrum among diverse applications and users. Can UWB share the radio spectrum with existing users? What frequency-related limits such as emission bandwidth and lower frequency limit should be imposed on UWB signals? Should limits be established for time-related characteristics such as pulse width and pulse repetition rate (PRR)? If UWB systems proliferate, what are the effects of the aggregate of independent UWB signals?

Measurements of Ultrawideband Signals

From over twenty UWB devices available to ITS, five were chosen to be fully measured. This selection represents a sampling of the various UWB signal waveforms in use. This group included communications and sensing devices that used pulse-position and on/off keying modulation methods, some did not incorporate pulse dithering, another used relative dither and yet another used absolute-time-base pulse dithering, one had gated pulse groups.

A very fast transient digitizer was used to capture the individual pulses directly in the time domain (in some cases a sampling oscilloscope was used) at the output of each device (a "conducted" measurement) and "in space" as measured by a known antenna (a "radiated" measurement). Figure ES.1 shows examples: (a) is a narrow impulse about 1.5 ns in length from one of the UWB devices, and (b) is a longer, very complex pulse shape about 15 ns in length from a different device. The former occupies about 3.5 GHz of spectrum and the latter about a fifth or sixth of that. Although some devices generate an impulse like that shown above in ES.1(a), when radiated by an antenna, the impulse may be changed quite dramatically. Figure ES.1(c) shows what the pulse shown in (a) is like after being radiated by an antenna designed to radiate UWB signals.

Measurements of the UWB signal power in various bandwidths were made using spectrum analyzers and it was determined that the measurement of the signal amplitude probability distribution (APD) is a very informative measurand. It shows, sometimes in a dramatic way, the general nature of a UWB source, whether it resembles Gaussian noise or very impulsive noise. Figure ES.2 shows two APD curves on a Rayleigh probability scale. Curve A, actually a straight line, represents a signal that is Gaussian distributed; while curve B is the APD for one of the UWB devices measured. Notice that the signal exceeds about -55 dBm for 1% of the time and exceeds -80 dBm for about 12% of the time that it is on. Both signals here are noise-like; the former, a Gaussian distribution, is a truly random signal and the latter is a highly impulsive signal.

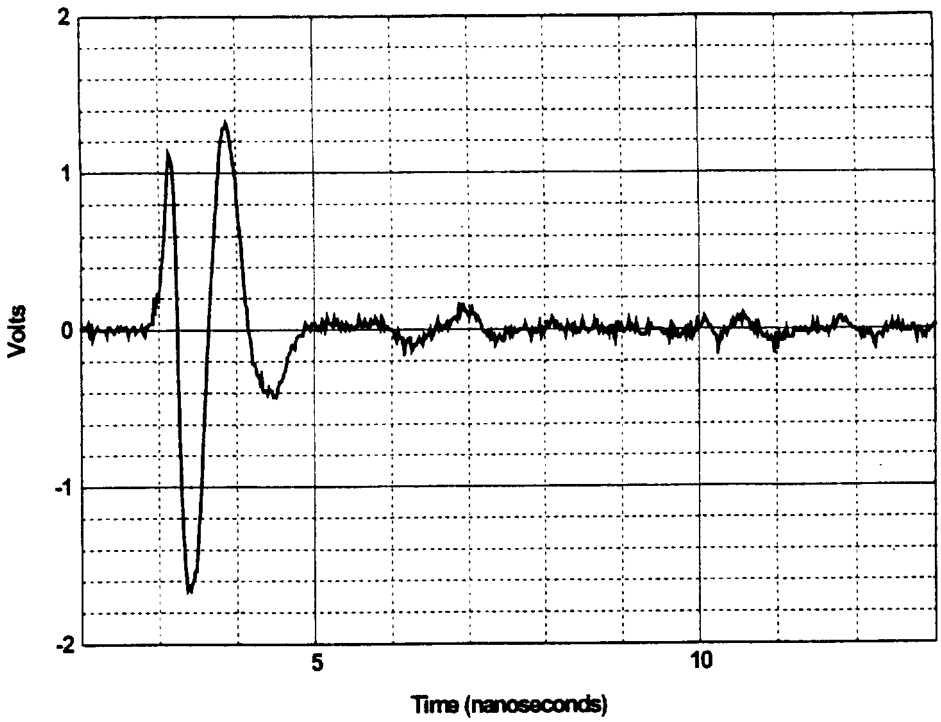

Figure ES.1(a). An example of a short UWB pulse.

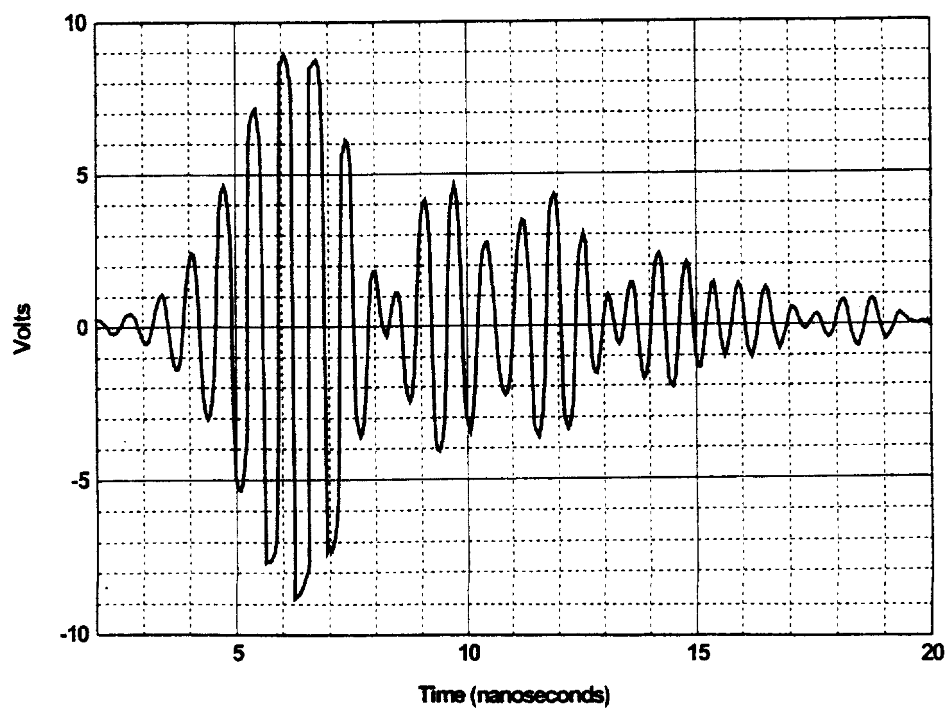

Figure ES.1(b). An example of a long UWB pulse.

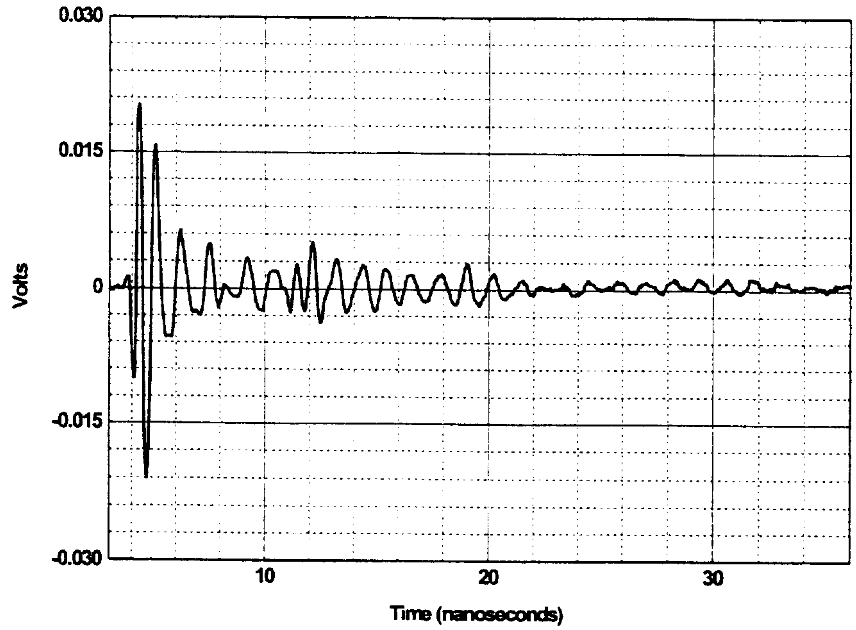

Figure ES.1(c). The shape of the pulse shown in (a) radiated, i.e., "in space."

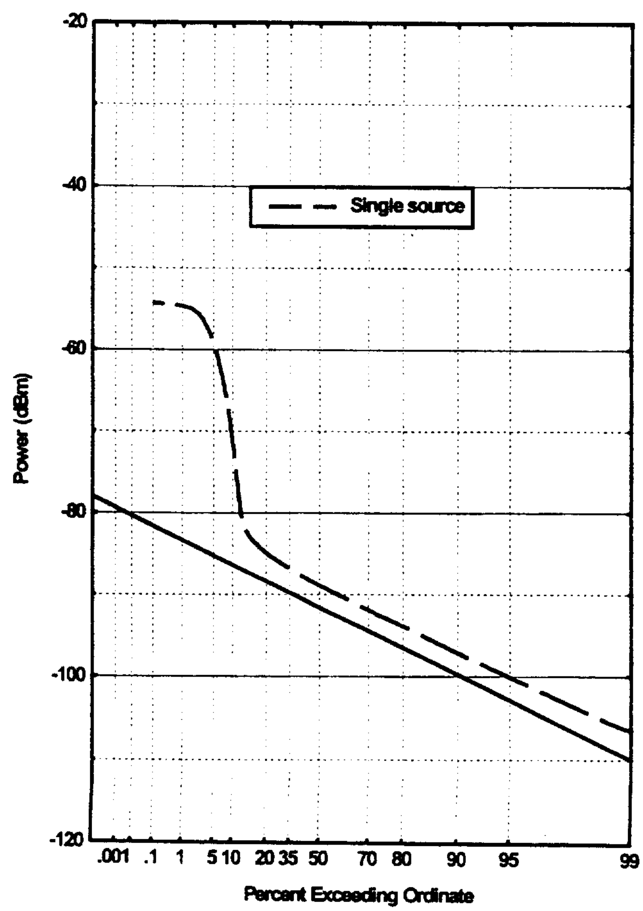

Figure ES.2. APDs for Gaussian noise and a UWB signal.

Companion Report and Other Investigations

The research, observations, measurements, and analyses presented in this report were performed at the NTIA Institute for Telecommunication Sciences (ITS), an independent laboratory located in Boulder, Colorado. The National Institute of Standards and Technology (NIST) Radio Frequency Technology Division, also located in the same building as ITS in Boulder, performed some of the measurements reported herein. The NTIA Office of Spectrum Management (OSM) has used the results of this investigation to examine the options and constraints appropriate for allowing UWB to share the use of the radio spectrum. In their report, a companion to this one, separation distances are developed for widely accepted receiver protection and interference criteria. It discusses the operation of UWB under unlicensed and licensed conditions.

A follow-on research effort at ITS has made use of the knowledge of UWB signal characteristics from this work to develop a test facility to measure the effects of UWB signals on Global Positioning System (GPS) receivers. The results of this GPS interference investigation will be published by ITS in a report similar to this one and OSM will use the GPS receiver performance data to determine the Federal government's position regarding the potential for UWB to share the spectrum, in particular, the GPS band at 1.5 GHz.

THE TEMPORAL AND SPECTRAL CHARACTERISTICS OF ULTRAWIDEBAND SIGNALS

William A. Kissick, Editor[1]

Ultrawideband (UWB) technology, useful for both communication and sensing applications, uses the radio spectrum differently than the vast majority of radiocommunication technologies. UWB systems make use of narrow pulses and time-domain signal processing. Questions regarding how these systems, with their potentially very wide emission bandwidths, might affect the efficient use of the radio spectrum or cause interference to conventional radio and wireless systems must be answered before there is any large-scale deployment of UWB systems. The investigation reported here examined both the temporal and spectral characteristics of UWB signals, since all radio signals exist in both the time and frequency domains. The investigation was approached with theoretical analyses, measurement of actual UWB devices, and computer simulations. The emissions of several UWB transmitters were measured under controlled, and repeatable, laboratory conditions. Those measurement methods useful for routine measurements using commercially-available test equipment were identified. The characteristics of an aggregate of several UWB signals were examined. An initial assessment of the effects of UWB signals on several Federal Government systems was accomplished through field measurements. This report provides a basis for an assessment of the effects of UWB signals on other communication and radar systems, the study of the spectrum efficiency of UWB technologies, and the development of spectrum sharing policies and regulations.

Key words: emissions, aggregate emissions, ultrawideband, UWB, time domain, frequency domain, radio spectrum, average power, RMS power, peak power, signal strength, pulse measurements, spectrum measurements.

1. THE RADIO SPECTRUM AND ULTRAWIDEBAND

William A. Kissick[1]

As the radio spectrum becomes more crowded due to the ever-increasing demand for radio and wireless communications and for sensing, a wide variety of creative approaches have been proposed for allowing more users to share this limited resource. These innovations include new, digital technologies that permit the same amount of information (e.g., an audio signal) to fit into increasingly narrower channels as is occurring in the land mobile radio service; or allow much more information to be transmitted in existing channels as is occurring with high-definition

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[1]The editor, and author of Section 1, is with the Institute for Telecommunication Sciences, National Telecommunications and Information Administration, U.S. Department of Commerce, Boulder, CO 80305.

television. This investigation is primarily concerned with one such approach called ultrawideband (UWB) technology and its ability to share the spectrum with existing users. There are claims that UWB technology, which uses novel signal generating and processing methods, can use large portions of the already allocated spectrum with minimal or no interference to existing users due to the very low spectral power density of UWB signals. The assessment of that claim is critical to decisions regarding the deployment, and potential ubiquitous use, of UWB devices for both communications and sensing. The radio spectrum, a nondepleting but limited natural resource, is used to support all radio and wireless services for both public and private purposes. Broadcasting, land mobile radio, cellular telephones, radar, satellite communications, remote sensing, and radio astronomy all depend upon the shared use of the radio spectrum which has benefitted mankind for the past century. The rules and regulations that enable this sharing are based on fundamental natural laws (of physics), agreements among proximate users, international treaties, and domestic public law. Spectrum management and frequency assignment represent the disciplines and processes used to allocate bands of the spectrum to various radio services and assign frequencies, each with an associated bandwidth, to individual users. In many cases, users are expected to ensure that their transmitter's emissions do not adversely affect existing users.

This approach is based on the fact that all electromagnetic signals can be both electronically generated and separated by the frequency of those signals. Essentially, all conventional signals use a single-frequency signal – a sinusoidal (sine) wave called a carrier that is modulated with the information it is to "carry." Its amplitude, frequency, or phase is varied according to the information (e.g., voice, video, or data) to be carried.

Radio and wireless communications are managed using tools and techniques that describe signals in the frequency domain; however, it is very important to recognize that every signal exists simultaneously in both the frequency domain and the time domain. These domains are simply alternative ways of describing and processing electronic and electromagnetic (radio) signals. This is easy to visualize. The cycling of a simple sine wave is the time domain perspective and its existence at a single frequency is the frequency domain perspective.

Using a carrier allows good control over the bandwidth any signal occupies and has been an enormously effective approach for dividing the radio spectrum by bands and channels, which has enabled tractable and effective sharing of the spectrum. It has always been possible, however, to generate signals without a carrier. In the case of UWB, these signals are simply pulses of electromagnetic energy shaped by electronic circuitry and a transmitting antenna. Recent advances in electronics and microcircuits have allowed the development of communications and radar systems that use such carrierless pulses. It is a fundamental physical law that the narrower the pulse in the time domain, the wider the emission in the frequency domain.

A classic radar is an example of a device that requires signal processing in both the frequency domain and the time domain. It has a carrier. Pulses of that carrier are transmitted periodically. Reflections of that signal from a target return to the point of origin. The radar receiver uses a filter in the frequency domain to select only that small portion of the spectrum where the radar signal exists. Then, the receiver uses time domain processing to determine how long it took the reflection to return and thus determine the distance to the target.

1.1 Objectives of this Work

The primary objective of this investigation was to develop an understanding of UWB signal characteristics based on several currently available devices. Both temporal and spectral characteristics of the UWB signals were sought, with the latter being of particular interest. The nature of the emission spectrum, whether smooth, comprised of lines, or a combination of both, is needed for interference analyses. How that emission spectrum depends on the temporal characteristics such as UWB pulse width, type of signal modulation, and the use of dithering may be needed to develop sharing policies and regulations. Finally, the nature of the aggregate of many individual UWB signals is important in understanding how the radio spectrum might be affected if and when large numbers of UWB devices are deployed.

Practical and repeatable measurement methods to obtain values for particularly useful UWB signal parameters with available commercial-off-the-shelf (COTS) test equipment may also be needed for compliance testing related to regulation. Where possible, these practical methods are identified and any limitations are described. Highly accurate time domain measurements are used to ensure that the COTS-based measurement methods are reliable.

Finally, an initial assessment of the effects of UWB signals on existing systems will indicate if, and how much, additional work is needed in this area. A limited effort to determine how much UWB signal power can pass through the front-end (antenna, amplifiers, and filters) of selected receivers provides a basis for more detailed investigations of the effects on victim system performance and allows the calculation of desired signal-to-noise and interference-to-noise ratios for the selected systems.

1.2 Specific Ultrawideband Systems Measured

The actual UWB emitters used in this work were borrowed from a number of sources, including UWB device manufacturers and owners of systems that contain UWB devices or that use UWB signals to perform their functions. These included prototype, experimental, and operational systems. Since the objectives of this work were to understand and characterize the radiated signals and not to evaluate the performance of the systems, the sources of UWB equipment are not identified. Of the dozen or so devices available, five were selected for the measurements described in subsequent sections of this report, and are labeled with letters, e.g. Device A. For comparison, the emissions of an electric drill were also characterized. It is identified simply as "electric drill." The devices selected are intended to provide a realistic sample of the various UWB signal structures being used today.

1.3 Organization of this Report

This investigation of UWB signal structure involved a number of aspects ranging from theoretical analyses to measurements, both in the laboratory and field, and computer simulations. As is often the case with broad investigations such as this, a number of workers with different skills were involved. To give proper credit to the researchers in each area, the author (or authors) of each major section of the report is identified at the beginning of each section. The editor was responsible for assembling the full report.

The first two sections provide orientation and background for the reader. This section contains some essential background information and the objectives of the work. Section 2 provides the reader with a brief technical description of UWB technology and some of its salient applications; this same section also gives a brief history of UWB development and early applications. It also contains a brief overview of the regulatory issues.

Sections 3 and 4 examine the UWB signal from first principles. Using typical temporal characteristics of UWB waveforms, the associated spectral characteristics are derived in Section 3. Then in Section 4, the characteristics of a group of individual UWB signals, called the aggregate signal, is examined.

Sections 5 through 7 describe a variety of measurements of UWB signals and their effects on selected receivers. Section 5 describes the procedures and results for fundamental measurements in the time domain. Where possible, the waveform of individual pulses is obtained. Section 6 describes procedures for, and results of, making similar measurements using commercially-available test equipment. This section also describes procedures for, and results of, band limited spectral measurements. Section 7 describes the effects that were observed in the receivers of several Federal Government systems. These measurements do not include an assessment of the overall performance of those systems; only those effects that are observable in the radio-frequency (RF) or intermediate-frequency (IF) sections of those systems.

Section 8 summarizes the observations made throughout this investigation. These observations include: the general character of UWB signals (spectra) based on theoretical analyses (Sections 3 and 4); the nature of the actual UWB pulses, both conducted and radiated (Section 5); the nature of the UWB signal in both the time and frequency domains when received in a range of bandwidths (Section 6); and the effects on selected receivers (Section 7). Other observations include which procedures may be best suited for other laboratories that may have only commercial-off-the-shelf (COTS) test equipment; and the effects various detectors have on measurements.

Section 9 contains a comparison of results from measurement, theory, and simulation.

The Appendices to this report contain supporting information and detailed measurement results. Appendix A is a brief tutorial on the amplitude probability distribution (APD) which was chosen as a key measurand for this work. Appendix B describes simulations on the UWB signal temporal and spectral characteristics of a UWB signal when passed through a limited bandwidth (receiver or test instrument). Appendix C describes how to convert and/or correct certain measured values. Appendix D contains the measured data for the five UWB devices and an electric drill. Appendix E contains the measured data for an aggregate of two, independent UWB signals.

2. UWB TECHNOLOGY AND REGULATORY ISSUES

William A. Kissick and Robert J. Matheson[1]

The term "ultrawideband" refers to the spectral characteristics of this technology and originates in the work that led up to a Department of Defense (DoD) study [1]. Alternative terms for the same technology include impulse radar, impulse radio, carrierless, carrier-free, time-domain, and others. The fundamental principle is that a short (in time) pulse, also called an impulse, is generated, transmitted, received, and processed. A fundamental principle, true for any radio signal, is the relationship between pulse duration and the bandwidth occupied by that signal.

According to the theoretical Fourier transform, a pulse of duration T seconds (in the time domain) has an occupied bandwidth of 2/T Hertz (in the frequency domain). For example, a pulse on the order of a nanosecond in the time domain occupies about two gigahertz of bandwidth in the frequency domain. An example of time domain signal processing is pulse-position modulation (PPM). Consider the transmission of a train of pulses equally spaced in time. The receiver processing determines whether each received pulse is located where expected or arrives early or late. With PPM, a slightly retarded pulse could represent a "0" and a slightly advanced pulse could represent a "1" when transmitting digital information.

2.1 History of Ultrawideband Technology

One could say that the first wireless[2] system demonstrated by Guglielmo Marconi in 1897 [3], meets the description of UWB radio. Marconi's earliest spark-gap transmitters occupied a large portion of the spectrum, from very low frequencies up through the high-frequency (HF) band and beyond. And, these systems used manual time domain processing. Morse code was sent and received by human operators.

The foundations of modern UWB systems were laid down in work done at the Sperry Research Center in the 1980's by Ross [4]. The emphasis was on the use of UWB as an analytical tool to explore the properties of microwave networks and to determine the intrinsic properties of materials [4,5]. These techniques were then logically extended to support experimental analysis and synthesis of antenna elements [6,7]. These early successes led to the development of an indoor system to measure the impulse response properties of targets or obstacles [8]. This

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[2]The term "radio" did not exist until 1912 [2]. It is a shortened form of "radioconductor" (a contraction of radiation conductor). "Wireless" was the common term before 1912.

approach of using "short-range radar" obviated the need for an expensive anechoic chamber to study radar targets, since unwanted reflections from walls and ceilings could be removed by time-gating techniques.

The use of UWB, with its time domain processing techniques, filled an important need in the early days of computer development. The appearance of high-speed, sub-nanosecond logic circuitry in the late 1960s and early 1970s made higher speed computation possible. However, it was necessary to deliver and distribute large amounts of digital data between the computer central processor and various input and output devices. This problem was solved by using multiplexing of multiple signals on a single transmission line using time-domain processing methods described in a patent by Ross, et al. [9]. This patent could be viewed as a key element in the foundation of UWB communications. It is a small step from this work to developing wireless UWB communications. Further developments during the 1970s led to a more thorough development of principles needed to fully describe and develop the field of time-domain electromagnetics [10, 11, 12].

In the 1980s and 1990s the principles of time domain electromagnetics were applied to wireless communications, in particular to short-range communications in dense multipath environments. Schotz [13] describes this application in detail and explores the advantages and disadvantages. He showed that a large number of such systems could operate in the same space and that such wide bandwidth signals are more immune to the deleterious effects of multipath than are narrow bandwidth signals. A potential application for UWB communications is the accommodation of many users in high-multipath environments, but the challenge is coexistence within the already highly-populated radio spectrum. The advantages may or may not outweigh the disadvantages, and other approaches to wireless operation in dense, high-multipath environments may perform as well as the UWB approach.

The other major application area of UWB technology is sensing, with the likely niche being short-range, high-resolution radar. This area requires much less signal processing and uses much simpler electronics, but has not received as much attention as the more complex communications applications. Ground penetrating radar was one of the first applications [14]. In 1974, Morey [15] patented a radar system that, due to the use of a very wide band of frequencies, was able to penetrate the ground to distances of one to several meters. This patent was later the basis of a commercial success.

2.2. Regulatory Issues

After receiving three requests by UWB developers, the Federal Communication Commission (FCC) issued a Notice of Inquiry (NOI)[3], to gather information on the possible uses of UWB devices. Many comments were received in response to that NOI. The FCC also issued the

[3]OET Docket 98-153, NOI issued Sept 21, 1998.

requested three waivers for a limited number of each of the three low power UWB devices after coordination on the technical limitations required by NTIA to approve the proposals[4]. Information gathered by that NOI led the FCC to release a Notice of Proposed Rule Making (NPRM) in May 2000. The major regulatory issues in that NPRM are centered on the question of how much interference UWB systems might cause to existing radio systems.

The FCC and NTIA jointly manage the radio spectrum in the United States. Part 15 of Volume 47 of the Code of Federal Regulations (47 CFR- Part 15) contains the FCC rules for authorizing non-licensed operation of low power radio devices that typically radiate signals in bands licensed for other types of devices. The current Part 15 rules define three classes of radiators: Incidental Radiators (which do not deliberately generate the RF signals they emit and are not regulated; e.g., an electric drill), Unintentional Radiators (which need to generate RF signals, but do not intend to radiate them, e.g., a computer), and Intentional Radiators (which deliberately radiate low-level radio signals, e.g., a garage door opener). The NPRM proposes that UWB devices be operated under a new section of the Part 15 rules, with approximately the same numerical limits for new UWB devices as for the existing intentional radiators.

Major regulatory issues include a determination of what numeric limits should apply to UWB emissions and what techniques should be used to measure those emissions. The NPRM proposes numerical limits and measurement techniques identical to those described in current Part 15 rules for Intentional Radiators, with the addition of a maximum total absolute peak limit or a possible peak limit measured in a 50-MHz bandwidth.

Important related questions include whether these limits should be lower in specific restricted frequency bands used by the Federal Government for particularly critical applications, including the Global Positioning System (GPS). These critical frequency bands have already been identified in the existing Part 15 rules, and Intentional Radiators are prohibited from deliberately radiating signals in any of these identified critical bands. Since UWB systems will typically radiate energy in frequency bands managed by NTIA, as well as frequency bands managed by the FCC, the two agencies must concur on the new rules.

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3. ANALYTICAL DESCRIPTION OF TIME AND SPECTRAL CHARACTERISTICS OF ULTRAWIDEBAND SIGNALS

Roger A. Dalke[1]

3.1 Introduction

A theoretical analysis of UWB signals can provide important insights into how UWB emissions affect various types of RF communications devices. In addition to allowing for direct calculation of interference effects, analytical results can be used to aid in the planning, design, and validation of measurements. This section details the results obtained from an analysis of proposed UWB pulse position modulation schemes.

The approach used in the analysis and the results are presented in this section. The mathematical details will be published elsewhere.

3.2 Power Spectrum of UWB Signals

The power spectral density is the average power in the signal per unit bandwidth and hence provides important information on the distribution of power over the RF spectrum. The power spectral density for a UWB pulse position modulation scheme using short duration pulses transmitted at some nominal pulse repetition rate (PRR) is given in this section. The pulse position is randomized or *dithered* with respect to the nominal pulse period. The randomization scheme analyzed in this section is referred to as *fixed time-base dither*.

3.2.1 UWB Signals Using Fixed Time-base Dither

In the fixed time-base dither scheme, each pulse occurs at the nominal pulse period, T, minus a time increment randomly distributed over a fraction of the nominal period as given in Equation 3.1. This expression also includes binary pulse modulation as proposed for communications applications.

$$x(t) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{1} \alpha_{kn} p_k(t - nT - \theta_n) \quad , \tag{3.1}$$

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where p_k represents the pulse shape that corresponds to an information bit (e.g., p_0 represents the value 0, p_1 represents the value 1). The coefficients α_{kn} are related to whether the n[th] information bit a_n has the value 0 or 1 as follows:

$$\alpha_{kn} = \begin{cases} 1 - a_n & k = 0 \\ a_n & k = 1 \end{cases} ,$$

$$a_n = \begin{cases} 0 \;\; with\ prob\ g_0 \\ 1 \;\; with\ prob\ g_1 = 1 - g_0 \end{cases} \tag{3.2}$$

where g_k are the information bit probabilities (i.e., g_0 is the probability of a bit having the value 0, and $g_1 = 1 - g_0$ is the probability of a bit having the value 1). Finally, the random variables θ_n define the pulse randomization or *dithering* and are described by a density function $q(\theta)$, where

$$\wp r\,\{\theta \le \Theta \le \theta + d\theta\} = q(\theta)\,d\theta \quad . \tag{3.3}$$

For fixed time-base dither, the random variables θ_n and a_n are each assumed to be independent and identically distributed (iid).

It should be noted that the signal given in Equation 3.1 is quite general in terms of the pulse shape, binary modulation method, and pulse randomization statistics. Hence, the results presented in this section can be used to predict the power spectral density at various points in the radio link between an interfering UWB transmitter and a victim receiver (e.g., at the output of the UWB transmitter, the UWB signal radiated from a particular antenna, or in the IF section of a *narrowband* RF receiver). When dealing with linear systems, the various pulse shapes are simply related by convolutions with the appropriate transfer functions.

The power spectral density is the Fourier transform of the autocorrelation function. The autocorrelation function is obtained by taking the expected value of the signal at two different times which is expressed mathematically as

$$\begin{aligned} r_{xx}(t,s) &= \mathscr{E}\{x(t)x(s)\} = \\ &\mathscr{E}\Big\{\sum_n \sum_m \sum_k \sum_\ell \alpha_{kn}\alpha_{\ell m}\, p_k(t - nT - \theta_n)\, p_\ell(s - mT - \theta_m)\Big\} \quad . \end{aligned} \tag{3.4}$$

Taking the expectation in Equation 3.4 yields

$$
\begin{aligned}
r_{xx}(t,s) = {} & \frac{1}{T^2}\Bigg\{\sum_{n}\Big|\sum_{k=0}^{1} g_k P_k(\tfrac{n}{T})\Big|^2\Big|Q(\tfrac{n}{T})\Big|^2 e^{i2\pi n\tau/T} + \\
& \sum_{n\neq -m}\Big(\sum_{k=0}^{1} g_k P_k(\tfrac{n}{T})\Big)\Big(\sum_{k=0}^{1} g_k P_k(\tfrac{m}{T})\Big) Q(\tfrac{n}{T}) Q(\tfrac{m}{T}) e^{i2\pi(nt+ms)/T}\Bigg\} \\
& + \\
& \frac{1}{T}\sum_{n} e^{i2\pi ns/T}\Bigg\{ Q(\tfrac{n}{T}) \sum_{k=0}^{1} g_k p_k(\tau)\circledast p_k(-\tau) e^{i2\pi n\tau/T} - \\
& \Big(\sum_{k=0}^{1} g_k p_k(\tau)\circledast\sum_{l=0}^{1} g_l p_l(-\tau) e^{i2\pi n\tau/T}\Big)\circledast\Big(q(\tau)\circledast q(-\tau) e^{i2\pi n\tau/T}\Big)\Bigg\}
\end{aligned}
\tag{3.5}
$$

were the symbol $\circledast$ is the convolution operator and $\tau = s - t$ is the time lag. Functions given in upper case letters (P, Q) are the Fourier transforms of the pulse and dithering functions.

The statistics for this process are periodic with period T as is evidenced by Equation 3.5. Such processes are commonly referred to as *cyclostationary*. Essentially this means that the statistics depend upon when the process is observed during a period. The victim receiver may observe the process at an arbitrary time during a period and hence it is useful (and simplifying) to calculate the average over all possible observation times within a period. Taking the time average over one period and the Fourier transform of Equation 3.5 yields the average power spectral density of the fixed time-base dithered UWB signal

$$
\begin{aligned}
& \bar{R}_{xx}(f) = L + C \\
& L = \frac{1}{T^2}\Big|\sum_{k=0}^{1} g_k P_k(f)\Big|^2\Big|Q(f)\Big|^2\sum_{n}\delta(f - n/T) \\
& C = \frac{1}{T}\Bigg[\sum_{k=0}^{1} g_k|P_k(f)|^2 - \Big|\sum_{k=0}^{1} g_k P_k(f)\Big|^2\Big|Q(f)\Big|^2\Bigg] \quad .
\end{aligned}
\tag{3.6}
$$

The power spectral density has both discrete L and continuous C components that depend on the pulse spectrum and the Fourier transform of the density function used to randomize the signal. Note that when $Q(f)$ is small at multiples of the PRR, the discrete components are small and the spectrum is predominantly continuous. When $Q(f)$ approaches one (negligible dithering) and the bits do not change (e.g., $g_0 = 1$), the continuous spectrum disappears, and the line spectrum dominates. The quantity $g_0 P_0(f) + g_1 P_1(f)$ is the expected value of the pulses.

If bit values are equiprobable (i.e., $g_k = 1/2$) and the pulse representing a 1 is a time delayed version of the pulse representing a 0 (i.e., $p_1(t+\xi) = p_0(t) \equiv p(t)$), equation 3.6 reduces to

$$
\begin{aligned}
&\bar{R}_{xx}(f) = L + C \\
&L = \frac{1}{2T^2} |P(f)Q(f)|^2 [1 + \cos(2\pi\xi f)] \sum_n \delta(f - n/T) \\
&C = \frac{1}{T} |P(f)|^2 \left(1 - \frac{|Q(f)|^2 [1 + \cos(2\pi\xi f)]}{2} \right) \quad .
\end{aligned}
\tag{3.7}
$$

When the information bit time delay ξ is small relative to the to the dithering delay (i.e., $\cos(2\pi\xi f) \approx 1$ over the range of frequencies for which $Q(f)$ is significant), the effects of pulse position modulation on the power spectrum are inconsequential.

The results of an example calculation using Equation 3.7 are shown in the following figures. For this example, the signal consists of a short-duration pulse (Figure 3.1) transmitted at a 10 MHz PRR. The dithered pulse position is random and uniformly distributed over 50% of the pulse period. In this calculation, it is assumed that the effects of information bit modulation are negligible over the frequency range of interest. The power spectral density over a frequency range of 1-5000 MHz is shown in Figure 3.2. The magnitude of the spectrum is normalized to the peak of the continuous distribution (at about 250 MHz). The Fourier transform of the density function for this example is $Q(f) = sinc(\pi f T/2)$. This function has nulls at frequencies equal to $2k/T$ ($k = \pm 1, \pm 2, \pm 3, \ldots$), hence the interval between discrete spectral lines is 20 MHz as shown in the figures. For frequencies above 20 MHz, the continuous spectrum is approximately the same as the pulse spectrum (i.e., $P(f)$). Figure 3.3 shows the discrete spectrum over a more limited range (800-1600 MHz) to highlight the individual spectral lines.

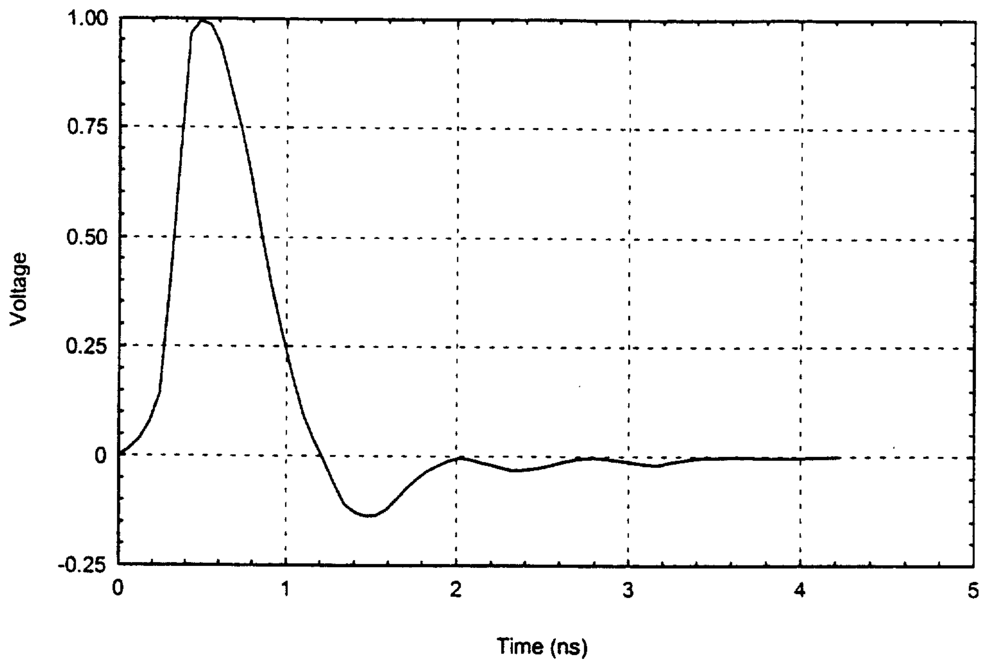

Figure 3.1. Time domain pulse shape.

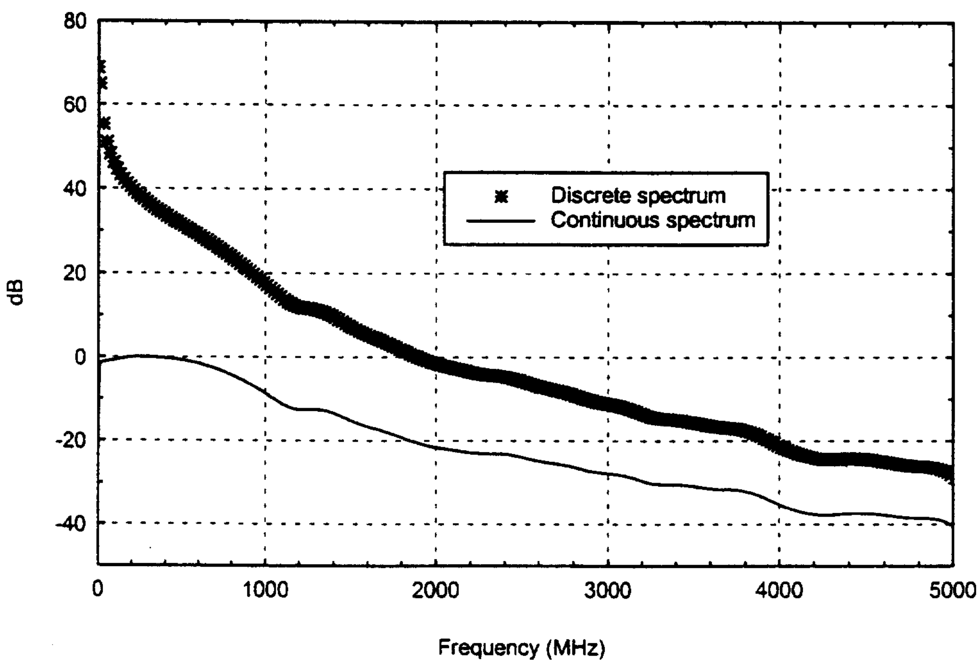

Figure 3.2. Power spectral density for a fixed time-base dithered 10 MHz UWB signal. The pulse positions are uniformly distributed over 50% of the pulse repetition period.

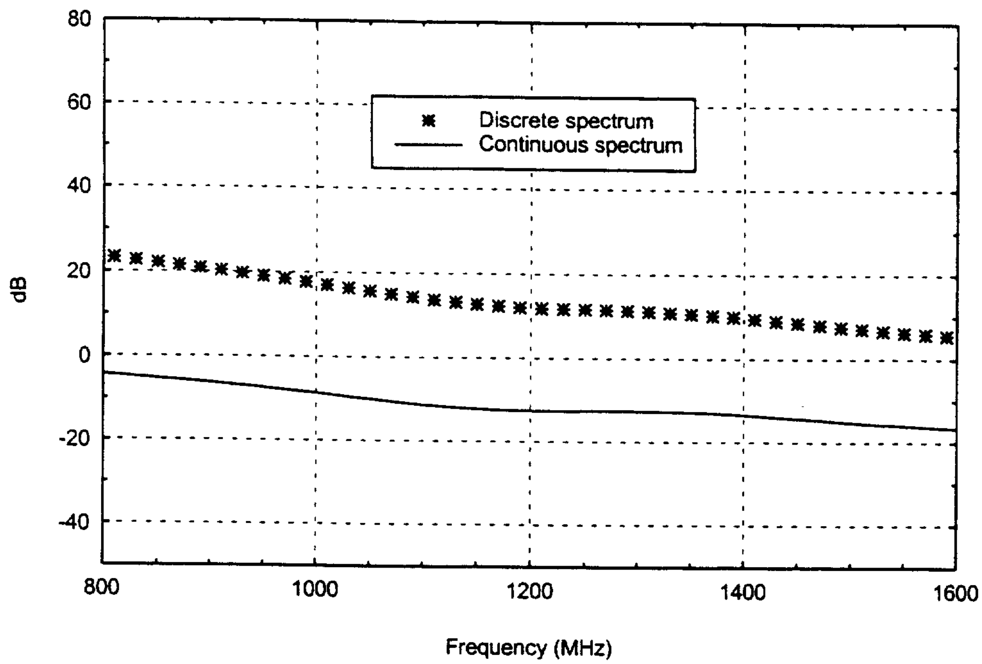

Figure 3.3. Power spectral density showing discrete and continuous spectrum from 800 to 1600 MHz.

The mean power in the bandwidth of a *narrowband* victim RF receiver as a function of frequency can easily be calculated from these results. For example, Figure 3.4 shows the power available to a receiver with a nominal 10 kHz bandwidth. As shown in the figure, the discrete spectrum is not a factor for RF frequencies above a few hundred MHz. For narrowband victim receivers where gains due to the UWB transmitter filters/antenna, propagation channel, and receiver are fairly constant over the receiver bandwidth, the received interference power can easily be calculated by applying the appropriate gain factors to the power in the receiver bandwidth at the center frequency of the receiver.

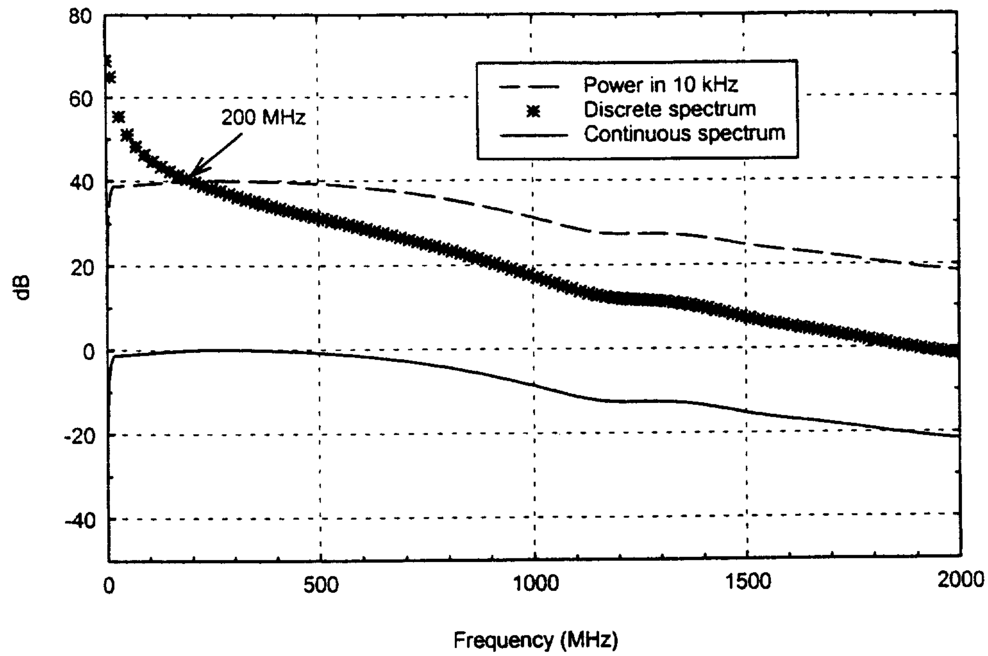

Figure 3.4 Power spectral density showing the continuous spectrum in a 10 kHz bandwidth compared to the discrete spectrum.

3.2.2 Power Spectrum for Finite Duration and Repeated Signals

The results based on Equation 3.1 assume that the signal is on continuously. Obviously, real signals are of finite duration. Also, for some proposed systems, the signal is transmitted for a length of time, say T', and then repeated. In this section, we extend the results presented above to finite duration and repeated signals.

To obtain the power spectrum for a finite duration signal, the following window function

$$w(t) = \begin{cases} 1 & -T' \le t \le T' \\ 0 & else \end{cases}$$
$$W(f) = 2T' sinc(2\pi T' f) \tag{3.8}$$

is multiplied by $x(t)$ (Equation 3.1). The result is that given in Equation 3.7 convolved with the spectrum of the window, i.e., $|W(f)|^2 \circledast \bar{R}_{xx}(f)$ as may be expected. As the window duration increases, the spectrum shape approaches $\bar{R}_{xx}(f)$.

When the series $x(t)$ is windowed and repeated, the autocorrelation function is obtained by taking the expectation of periodic extension of a windowed portion of the series or

$$\mathscr{E}\left\{\sum_{n=-\infty}^{\infty} w(t-nT')x(t-nT')\sum_{m=-\infty}^{\infty} w(s-mT')x(s-mT')\right\} \tag{3.9}$$

The resulting spectrum is

$$\frac{1}{T'^2}\sum_k \bar{R}_{xx}(\frac{k}{T'})\circledast|W(\frac{k}{T'})|^2 e^{i2\pi k\tau/T'} \quad , \tag{3.10}$$

which is now discrete with *spectral lines* at frequency intervals of $1/T'$.

3.3 Band Limited Signal Statistics for Fixed Time-base Dithered Systems

From the standpoint of a victim receiver, a fixed time-base dithered UWB signal is a random process. A knowledge of the statistics of such a process is important in predicting how interference affects the performance of a victim receiver. When the UWB PRR is larger than the receiver bandwidth, it may be expected that the received signal would appear to be indistinguishable from Gaussian noise. Since receiver performance in a Gaussian noise environment is well understood, quantifying conditions for which the received UWB interference resembles Gaussian noise is important in predicting receiver performance and developing emissions requirements. Also, when the received signal is Gaussian, only one parameter (mean power) is required to characterize the process. In this section we present the results of an analysis of the fixed time-base dither scheme that can be used to predict when the received UWB signal is approximately Gaussian.

For this analysis, we seek to determine the probability density function that describes the statistics of the UWB signal as seen by the victim receiver (e.g., the final IF stage of the receiver). The following relationship between the density function $a(y)$, its characteristic function $\phi(u)$, and the pulse randomization density function $q(\theta)$ is used to obtain an approximate expression for the received signal statistics

$$\phi(u) = \int e^{iuy} a(y)dy = \mathscr{E}\{e^{iux}\} = \int e^{iux(\theta)} q(\theta)d\theta \quad . \tag{3.11}$$

Formally, the desired density function is obtained by inserting the UWB signal $x(t)$ (Equation 3.1) into Equation 3.11 and taking the inverse Fourier transform of the characteristic function.

The characteristic function is periodic since the process is cyclostationary as discussed in Section 3.2.1. For purposes of this analysis, the time averaged statistics are obtained by averaging over a period as with the power spectral density function

$$\bar{\Phi}(u) = \int_0^{T_0} \prod_n \int e^{iup(t-nT-\theta)} q(\theta) d\theta \frac{dt}{T} \quad . \tag{3.12}$$

After some manipulations, the density function can be expanded into the well known Edgeworth [1] series. The first four terms of the series are

$$f(x) = \varphi^{(0)}(x) - \frac{\gamma_1}{3!}\varphi^{(3)}(x) + \frac{\gamma_2}{4!}\varphi^{(4)}(x) + \frac{10\gamma_1^2}{6!}\varphi^{(6)}(x) \quad , \tag{3.13}$$

where

$$\varphi^{(n)}(x) = \frac{d^n}{dx^n}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} \quad . \tag{3.14}$$

The desired density function $a(y)$ is related to $f(x)$ by using the transformation $x = (y-m)/\sigma$ where m is the mean and σ is the standard deviation, hence $a(y) = f((y-m)/\sigma)/\sigma$. The first term in the series is the standard normal distribution. The following terms are scaled by coefficients known as the skewness γ_1 and excess γ_2 [1].

In general, the skewness and excess are rather complicated functionals of the pulse shape p and the pulse randomization statistics q. In the case of a narrowband receiver with a center frequency larger than twice the PRR, the expressions are greatly simplified. The following results assume that the power in the spectral lines (if present) is much smaller than that due to the power in the receiver bandwidth due to the continuous spectrum. In addition, if the UWB pulse $P(f)$ spectrum is approximately constant over the bandwidth of the receiver, the variance σ^2, skewness, and excess can be expressed in terms of the baseband impulse response of the receiver filter, $h(t)$, as follows:

$$m \approx 0$$

$$\sigma^2 \approx \frac{1}{2T}\int_{-\infty}^{\infty} h^2(t)\,dt = \frac{1}{2T}\int_{-\infty}^{\infty} |H(f)|^2\,df$$

$$\gamma_1 \approx 0 \qquad (3.15)$$

$$\gamma_2 \approx \frac{3}{4\sigma^4 T}\int_{-\infty}^{\infty}\left[\frac{h^4(t)}{2} - (h^2 \circledast q(t))^2\right]dt$$

These results show that the variance is proportional to the receiver bandwidth as expected. The mean and skewness are negligible due to the oscillatory characteristics of the bandpass filtered signal. The behavior of the excess as a function of receiver bandwidth was calculated for a receiver with a raised cosine lowpass characteristic and a UWB signal with a 10 MHz PRR. The signal is dithered uniformly over 50% of the pulse repetition period.

Figure 3.5 shows the excess as a funtion of receiver bandwidth. Note that the distribution is approximately Gaussian up to about a 1MHz bandwidth. The excess then decreases to a minimum at about 20 MHz, after which it increases. The normalized distribution for bandwidths below 1 MHz and at10 and 20 MHz are shown in Figure 3.6.

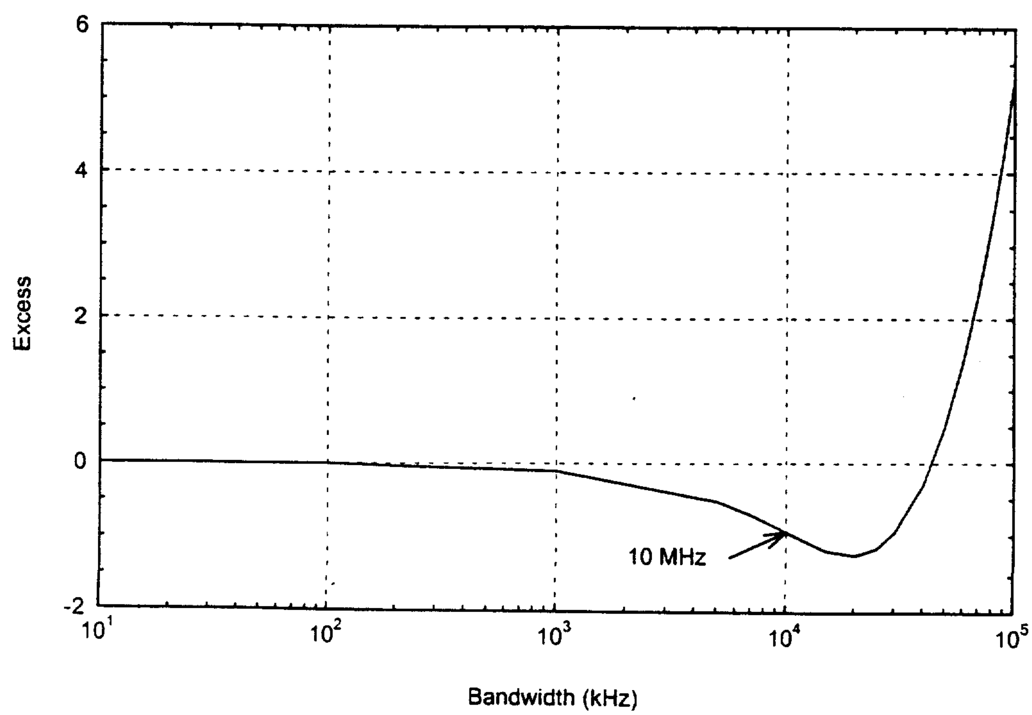

Figure 3.5. The excess as a function of receiver bandwidth.

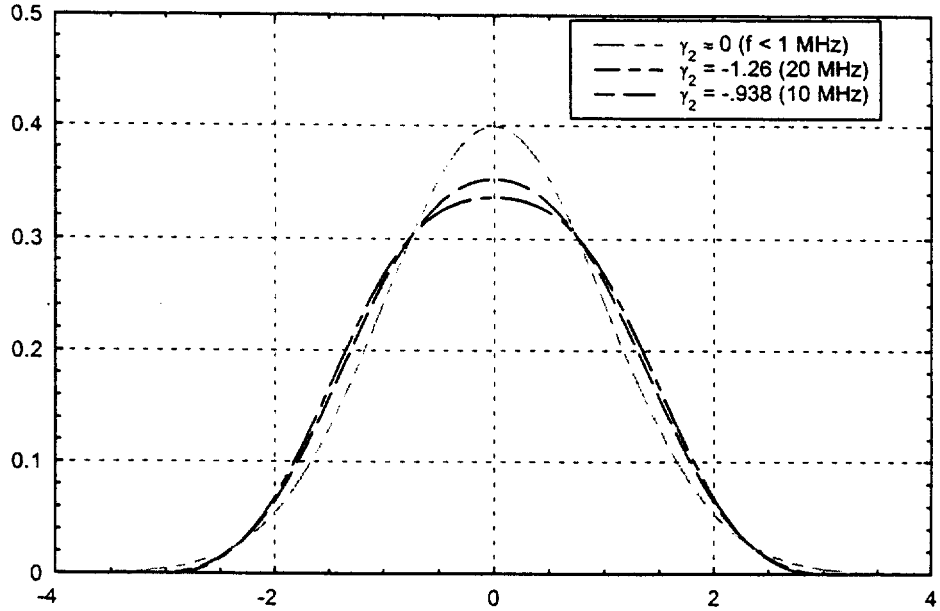

Figure 3.6. The distributions for various receiver bandwidths of less than 1 MHz, and bandwidths of 10 and 20 MHz.

The results presented in this section can be used to predict when an interfering fixed time-base dithered UWB signal is approximately Gaussian in nature, and hence, should be useful in providing guidance to system designers and regulators. Furthermore, as shown in the previous example, they can be used to estimate statistics for bandwidths comparable and exceeding the UWB PRR. In cases where the bandwidth is much larger than the PRR, so that the receiver actually resolves the individual pulses, the results presented above are no longer valid. In such cases, amplitude statistics can readily be estimated by calculating the fraction of time that a particular pulse (as seen by the receiver) amplitude is exceeded during the pulse repetition period.

3.4 References

[1] Harald Cramer, *Mathematical Methods of Statistics*, Princeton NJ: Princeton University Press, 1945.